# Supporting Information S1-S6: Finding the best management policy to eradicate invasive species from spatial ecological networks with simultaneous actions.

## Supporting Information S1. Species Dynamics Model description

**MDP definition**

*States*: where , and , . After maximum likelihood fitting, we impose the simplification because RFBE and mosquitofish are unlikely to coexist.

*Actions*: , corresponding to: do nothing, poison pool respectively. Potentially 2 actions per pool.

*Rewards*: Reward for pool *i*: where is the relative value of having RFBE present in a spring and is the relative cost of having mosquitofish present in a spring. For this model we set so that the value of a spring occupied with RFBE is 10 units and the value of a spring occupied with mosquitofish is -10 units. is the cost of action A in spring i at time t. Cost and .

*Discount factor:* The discount factor γ=0.95 determines the relative value of immediate rewards compared to future rewards. Selecting γ close to 1 means that immediate and future rewards have similar value in the optimisation.

**Transition model:**

Our model allows at most one event (i.e. colonisation or mortality, but not both). We define transition probabilities for the invasive species *M* as:

Let

 be the probability of colonisation of patch i from patch j.

 be the baseline mortality rate for patch i, assuming no action.

be the probability that poisoning successfully eradicates mosquitofish

 be the probability that any patch i is recolonised from a source population

**FOR THE INVASIVE SPECIES (Mosquitofish):**

**DO NOTHING ACTION: (a=0)**

**POISON ACTION: (a=1)**

 =

**FOR THE ENDEMIC SPECIES (Red-Finned Blue-Eye):**

We define transition probabilities for the endemic species *R*, defined as:

Let

 be the probability of colonisation of patch i from patch j given that mosquitofish are not present in patch i.

 be the probability of colonisation of patch i from patch j given that mosquitofish are present in patch i.

 be the baseline mortality rate for patch i, assuming that mosquitofish are not present.

 be the baseline mortality rate for patch i, assuming that mosquitofish are present.

be the probability that poisoning successfully eradicates the endemic RFBE fish

**DO NOTHING ACTION: (a=0)**

**POISON ACTION: (a=1)**

(Note that after parameter estimation, we found that the probability that RFBE and mosquitofish cohabit the same spring was very low (see in Table 1 of main text; note that that ). Consequently, we reduced the number of model states by assuming = 0, i.e. we set and ).

**MODEL PARAMETERISATION: MAXIMUM LIKELIHOOD**

Our model assumes that colonisation is a dispersal function with one parameter that links colonisation to distance between pools *i* and *j*. We also assume that mortality is a function of the area of pool *i*. Model parameter is assumed to be known for the purposes of likelihood estimation.

The parameters that need to be estimated for the ***invasive species*** are:

Where is the log of the likelihood function for the invasive species in terms of parameters

The likelihood function is defined by the probabilities of transition:

 , where , and the data series is 14 years long.

Because the initial state is known from the data (assuming perfect detection), then for all pools *i*, so the likelihood is:

The log likelihood for the invasive species is therefore:

We use an optimisation algorithm (optim function in program R; using method “L-BFGS-B”) to solve for values of parameters that maximise .

The parameters that need to be estimated for the ***endemic species*** are:

Where the parameters describe the mortality and colonisation of the endemic species in the absence and presence of the invasive species respectively. These have the same relationships to distance and area as in the invasive species model, however a different parameter will be estimated because the transitions are different (see above for transition model for endemic species in presence and absence of invasive species).

Using the same reasoning as above, the log-likelihood for the endemic species is:

For both the endemic and invasive species, we varied the initial parameter values to search for an improved local maximum using combinations of a range of plausible initial parameter values. Plausible values were selected using a two step process. First, a set of parameter values were selected at random and were used to run the likelihood optimization until a local maximum was reached. Second, another set of initial values was selected by choosing new initial values above and below the first set of converged parameter estimates. This second set of parameter values was used as initial values in the likelihood optimization-- by starting from a different combination of initial values, we increase the chance of finding an improved local maximum in the likelihood function. For the mosquitofish, the initial parameter values tested were: = 1,5,9,20; = 0, 0.0001, 0.001, 0.01 (total 16 combinations). For the RFBE, the initial parameter values tested were: = 10, 20, 30, 40; = 10,25,50,75; = 0, 0.01, 0.005, 0.001; =0, 0.1, 0.2, 0.3 (total 256 combinations). The best parameter values found are reported in Table 1 of the main text.

Confidence intervals for the parameter estimates were generated by resampling the dataset using a leave-one-out approach. We generated 14 subsets of the data, leaving out one timestep of data in each subset, then computed maximum likelihood parameters from each training subset. The resulting parameter distributions were used to compute 95% confidence intervals for the mean, i.e. using , where is the mean parameter estimate, s is the sample standard deviation, and is the critical value of the t-distribution with n-1 degrees of freedom.

## Supporting Information S2. Species dynamics simulations under the GMDP policy

This section contains simulations of the optimal policy when the probability of eradication success is high (Figure S1, ) and low (Figure S2, ) respectively. The rule of thumb for management is similar in both cases, however when the eradication probability is low, the rule depends on the strength of the connectivity between springs (Figure S2; e.g. E518 is not eradicated when mosquitofishare present in SW60 because the probability of colonisation between these springs is very low, 0.06)



Figure S1: Example simulation of the species dynamics model for three sub-networks (left column) with high management success (), following the GMDP policy. A good policy (right column) is to eradicate mosquitofish in all occupied nodes and their immediate neighbours, unless RFBE are present in the neighbouring nodes.



Figure S2: One simulation of the optimal policy with low management success (). The optimal strategy is to eradicate mosquitofish in all occupied patches and their immediate neighbours, unless RFBE are present in the neighbouring patches.

## Supporting Information S3. Value of information analysis and performance against a heuristic

Uncertainty in the colonisation parameter of mosquitofish may result in conflicting optimal management policies. If uncertainty affects the optimal policy, then it is critical to know the true value of the colonisation parameter accurately because an incorrect colonisation parameter may lead to suboptimal management performance. The value of resolving the uncertainty in the colonisation parameter can be computed using the expected value of information approach ([Runge, Converse & Lyons 2011](#_ENREF_2)). The expected value of information can be computed using:

Where U(π,α) is the value or utility associated with policy π and colonisation parameter value α. The first term of the equation describes the expected utility given that uncertainty is resolved (i.e. the value of the GMDP policy given that we know the exact value of the colonisation parameter). The second term gives the expected utility given that we are uncertain about the value of the colonisation parameter (i.e. the average value of the three GMDP policies associated with the three colonisation parameter scenarios, weighted by their probability of occurrence). The difference between these two terms is the difference in performance that we would expect if we resolved the uncertainty about the colonisation parameter.

We use the optimal policy to compute the expected increase in performance if uncertainty was resolved, compared to the current level of uncertainty. If the expected value of resolving the uncertainty is less than the cost of monitoring to resolve the uncertainty, then monitoring should proceed. If not, then management may continue without resolving the uncertainty, as the management cost incurred by using the wrong colonisation parameter is less than the cost to resolve it.

In this case, resolving the uncertainty in the three colonisation probability scenarios (wet, no change or dry) provides no better expected performance than acting under uncertainty (number of springs occupied by mosquitofish, Table S1; and expected reward, Table S2).

To ground-truth the optimal solution, each GMDP policy was tested against a simple heuristic. The heuristic rule was to eradicate mosquitofish wherever they occur, and do nothing otherwise. In all scenarios the GMDP solutions significantly out-performed the heuristic. The expected performance assuming randomly chosen actions is also reported (Table S1; Table S2).

Table S1: The expected value of resolving the uncertainty in the three colonisation probability scenarios (wet [-10% change], current conditions [no change], or dry [+10% change] in colonisation parameter). The correct scenario may not be known (here assumed that each scenario will occur with probability 0.33) but a management decision must be made. Management will either be correct (e.g., manage as if the colonisation parameter is increasing when it is) or incorrect (e.g., manage as if the colonisation parameter is increasing when it is not). The top half of the table shows the average (derived from 100 simulations) mosquitofish occupancy in the spring network accumulated over 25 years for correct and incorrect management scenarios. These data are used to compute the expected value of perfect information; i.e., the additional occupancy expected if the correct colonisation parameter is known. In the last row, the expected value of information is converted to the expected percentage increase in performance if the correct colonisation scenario was known. The expected increase in performance from knowing the correct inflow scenario was 0%.

|  |  |  |
| --- | --- | --- |
| TRUE MODEL | Probability model is correct | Average occupancy over 20 years, derived from 50 simulations |
| Manage as if wetter (0.9α) | Manage under current conditions (α) | Manage as if drier (1.1α) | Manage with heuristic | Manage at random |
| Wetter (0.9α) | 0.33 | 3.16 ± 0.02 | 3.22 ± 0.02 | 3.16 ± 0.02 | 3.72 ± 0.02 | 4.88 ± 0.04 |
| Current conditions (α) | 0.33 | 3.15 ± 0.02 | 3.19 ± 0.02 | 3.15 ± 0.02 | 3.59 ± 0.02 | 4.73 ± 0.04 |
| Drier (1.1α) | 0.33 | 3.19 ± 0.02 | 3.26 ± 0.02 | 3.18 ± 0.02 | 3.89 ± 0.03 | 5.03 ± 0.04 |
| Expected average occupancy if true model is unknown | 3.17  | 3.22 | 3.16 | 3.73  | 4.88 |
| Expected average occupancy if true model is known | 3.16 |
| Expected value of perfect information | 3.16-3.16=0 |
| % occupancy gained from knowing correct model | 0% |

Table S2: The expected value of resolving the uncertainty in the three colonisation probability scenarios (wet [-10% change], current conditions [no change], or dry [+10% change] in colonisation parameter). The correct scenario may not be known (here assumed that each scenario will occur with probability 0.33) but a management decision must be made. The top half of the table shows the average (derived from 100 simulations) expected rewards accumulated over 25 years for correct and incorrect management scenarios. The expected increase in performance from knowing the correct inflow scenario was 0%.

|  |  |  |
| --- | --- | --- |
| TRUE MODEL | Probability model is correct | Average occupancy over 20 years, derived from 50 simulations |
| Manage as if wetter (0.9α) | Manage under current conditions (α) | Manage as if drier (1.1α) | Manage with heuristic | Manage at random |
| Wetter (0.9α) | 0.33 | -430 ±220 | -436 ±223 | -432 ±220 | -529 ±235 | -1487 ±172 |
| Current conditions (α) | 0.33 | -414 ±215 | -413 ±216 | -415 ±214 | -482 ±228 | -1451 ±170 |
| Drier (1.1α) | 0.33 | -461 ±217 | -468 ±222 | -460 ±218 | -602 ±242 | -1525 ±181 |
| Expected average reward if true model is unknown | -435 | -439 | -436 | -538 | -1488 |
| Expected average reward if true model is known | -434 |
| Expected value of perfect information | -434-(-435)=1 |
| % reward gained from knowing correct model | 0% |

## Supporting Information S4. Effectiveness of a one-off treatment

Although taking simultaneous actions can result in fewer springs occupied by mosquitofish, taking multiple actions per timestep requires many more eradication actions over time than acting in a single node. We tested a compromise solution, where simultaneous actions were applied in the first timestep, resulting in multiple mosquitofisheradications. After this initial treatment, no action was taken for the rest of the simulated management period. The shock treatment has a very similar total cost (24 units, all in year 1) to the one action per time step policy (25 units, distributed evenly over 25 years). The shock treatment was initially very effective, however over the simulated period, mosquitofish populations recovered and the shock treatment was little better than doing nothing (Figure S3).



Figure S3: Comparison of the one action per time step solution and a 'shock treatment'. In the shock treatment, the policy recommended by the GMDP is followed for the first time step and no action is taken for the remainder of the management period. Results are shown for 100 simulations of 25 years; error bars represent 1 standard deviation.

## Supporting Information S5. GMDP effectiveness on large networks

Finding the GMDP policy for the Edgbaston network was difficult because network nodes had more than 5 neighbours (the effectiveness of the GMDP algorithm relies on each node having few neighbours). To obtain a solution to the problem, we reduced the number of edges by applying a connectivity threshold (0.05), which had the effect of disconnecting the network into subgraphs which were solvable in a reasonable time using the GMDP. However, by applying the connectivity threshold in our case study, the number of nodes in each subgraph was sufficiently reduced so that the disconnected case study network could in principle be solved using a standard MDP solver.

To illustrate the benefits of the GMDP for finding policies for larger networks with fewer connections, we constructed the Edgbaston network without a connectivity threshold, then removed all edges at each node except the edge with highest colonisation probability. We did this for the mosquitofish network, the RFBE network in the presence of mosquitofish, and the RFBE network in the absence of mosquitofish. We then selectively removed and added edges to obtain a fully connected network where each node had no more than 3 neighbours (Figure S4; Table S3). Added edges were assigned a connectivity probability of 0.01.



Figure S4: Constructed Edgbaston network with selectively added and removed edges to facilitate policy generation using the GMDP. No connectivity threshold was applied to the network and no node in the network has more than 3 neighbours. Added edges were assigned a probability of connectivity of 0.01.

Although the constructed network does not represent the real network due to the artificially added and removed edges, it illustrates the power of GMDP for finding policies for large, weakly connected network problems. The network in Figure S5 contains 32 nodes and has a state space (332) that is too large to be solved by conventional MDP methods. Previous studies have demonstrated that the GMDP performs effectively on these kinds of networks ([Sabbadin, Peyrard & Forsell 2012](#_ENREF_3); [Cheng *et al.* 2013](#_ENREF_1)), and we verified that the GMDP solver converged for this network (we used a success probability of probability for this experiment). The GMDP solver found a solution in approximately one hour (3.6061x103 seconds). We attempted to construct an equivalent MDP version of the network to compare the performance with the GMDP, however the required transition matrix was too large to be stored in memory (required dimension is 332 x 332), so we were unable to generate an optimal policy.

Table S3: List of edges manually added and removed to construct network in Figure S5. Edge names refer to the two nodes connected by the edge. Added edges were assigned colonisation probability 0.01.

|  |  |
| --- | --- |
|  | Edge name |
| Edges added | E502-E518E525/524-E509E522-E501SE10-SE20SW42-SE40 |
| Edges removed | E502-NW90sSE60-SW42/40NW90s-SW60E525-SW60SW60-SW10SW50-SE10SW50-SE60 |

## Supporting Information S6. Weighted classification trees

The classification tree presented in the manuscript implicitly assumes that all states occur with equal probability. In reality, some states are more likely to be visited than others, which means that the error rate is likely to be biased unless all states are visited with equal probability. In this section we provide and demonstrate a method to generate a classification tree in which states are weighted by their relative probabilities.

The relative probability of occurrence of states in a Markov process at equilibrium follows the stationary distribution of the Markov process (when we solve a Markov decision process in infinite time, we use this property to report a stationary optimal policy). The stationary distribution of a Markov decision process can be found by first applying the optimal policy for each state to its corresponding row of the transition matrix to generate matrix , then solving the matrix equation:

Where is the stationary distribution of the Markov process.

The stationary distribution provides the probability of selection of each state and can be provided to the package rpart using the ‘weights’ option (for more information, see the rpart documentation: https://cran.r-project.org/web/packages/rpart/rpart.pdf). Rpart then applies case weights to remove the selection bias of the unweighted classification tree.

Like the unbiased classification tree, applying this procedure to the Edgbaston reserve case study resulted in a tree with very good performance relative to the GMDP solution (Figure S5). The weighted tree produces a better error rate than the unweighted tree using only the first split (for one of the largest subgraphs containing RFBE, the root node of the weighted tree misclassifies only 1.2 cases out of 234 cases; the unweighted tree misclassifies 106 out of 234 cases). Both trees give negligible error rates (0%) when they include all variables selected by the classification trees. However the weighted classification tree does not require that springs containing RFBE are never poisoned, which would potentially lead to rapid extinction of the RFBE. Because the unweighted solution gave a more acceptable classification tree, we did not include the weighted tree in the main text.

The reason that the weighted tree does not recognise the important rule (i.e. that springs containing RFBE should never be poisoned) is that RFBE become virtually extinct when the system reaches the stationary distribution (e.g. in the two largest subgraphs, maximum weight for any state where RFBE are present in the stationary distribution is 0). This is consistent with our findings that the outlook for RFBE is bleak unless additional management action is taken that targets RFBE expansion beyond its current range. It also emphasises the importance of not over-relying on automated solutions for rule of thumb generation—modellers and managers should always check the recommended solutions to ensure that they make intuitive sense as well as predicting low error rates.



Figure S5: A classification tree for selecting the best management action (a) performs similarly to the GMDP policy (b). In b, the mean occupancy of each species is computed from 200 simulations for a 50 year time horizon. Shaded regions depict one standard error. In both simulations the probability of eradication success is 0.9.

## References

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